Epistemology and Oligopoly Theory

Steven O. Kimbrough

Frederic H. Murphy

Yves Smeers

What do the players know?

- During Watergate: "what did he know and when did he know it?”
- Same question applies to firms in an oligopoly
  - Consistent conjectural variations
  - Allaz Vila
  - Tit for tat
  - Full knowledge of the equilibrium
- Logical contradiction of static equilibrium analysis

Let

z_i be the quantity produced by player *i*.

z_{-i} be the quantity produced by player -*i*.

$c_i(z_i) = \nu_i z_i + \frac{1}{2} \mu_i z_i^2$ be the cost of production

α be the intercept of the linear demand curve with slope -1.

The equilibrium condition coming from the optimization by player *i* is

$$\alpha - \left(2 + \frac{\partial z_{-i}}{\partial z_i}\right) z_i - z_{-i} - \nu_i - \mu_i z_i = 0$$

Solving for z_i, we get

$$z_i = -\frac{z_{-i} + \nu_i - \alpha}{2 + \mu_i + \frac{\partial z_{-i}}{\partial z_i}}$$

The derivative is

$$\frac{\partial z_i}{\partial z_{-i}} = -\frac{1}{2 + \mu_i + \frac{\partial z_{-i}}{\partial z_i}}$$

Let

$$r_i = \frac{\partial z_i}{\partial z_{-i}}$$

Then we have the following continued fraction

$$r_i = -\frac{1}{2+\mu_i - \frac{1}{2+\mu_i + r_i}}$$

Assuming symmetry in μ_i,

$$r_i = \frac{-2-\mu_i \pm \sqrt{(2+\mu_i)^2 - 4}}{2}$$

$$= \frac{-2-\mu_i \pm \sqrt{\mu_i^2 + 4\mu_i}}{2}$$

If $\mu_i = 0, r_i = -1$ and the equilibrium with consistent conjectural variations is the competitive/Bertrand equilibrium.

Implication of This Result

- The Kreps and Scheinkman (1983) model shows that in a two-stage game with a Bertrand spot market and a capacity game the equilibrium is Cournot.
- With consistent conjectural variations, Bertrand remains Bertrand.

Forward-Market Equilibria in Allaz Vila

With the linear costs of Allaz and Vila the equilibrium condition becomes

$$\frac{\partial z_i}{\partial z_{-i}} = -\frac{1}{2 - \frac{\partial z_{-i}}{\partial z_i}}$$

Starting with the slope of the reaction curves in the Cournot game, -1/2, and substituting in (12),

$$\frac{\partial\, z_i}{\partial\, z_{-i}} = -\frac{1}{2-\frac{1}{2}} = -\frac{2}{3}$$

Next we get

$$\frac{\partial\, z_{-i}}{\partial\, z_i} = -\frac{1}{2-\frac{2}{3}} = -\frac{3}{4}$$

The underlying formula is $\frac{-n}{(n+1)}$, which is -1 in the limit.

Comments on Allaz Vila

- This is the same proof as used by Allaz and Vila in the n-stage version. Each player sees the -1/2 slope to the reaction function of the other player when in the forward game, yet forgets the other player reacts in the spot game.
- If we impose consistent conjectural variations first, the solution is competitive and forward markets do not make a difference.
- This is a Tale with Two Stories for the same equations.
- Which can be believed?

Supply Curve Equilibria

- Are there equivalent issues with supply function equilibria?
- We don't know but suspect so.

Problems with Consistent Conjectural Variations

- If firms have full knowledge, why would they shoot themselves in the foot?
- Lindh (1992) points out that firm i knows how firm -i reacts but presumes *i* does not see how i reacts to -i actions. That is, they don't know the ultimate equilibrium consequences of their actions (no full knowledge)
- What happens when we include that knowledge?

Tit for Tat

- Find the equilibrium that comes from using a conjectural variation of 1 for both players
- For i =1,2 $\alpha - (2 + 1)z_i - z_{-i} - \nu_i = 0$
- $z_i = \frac{2\alpha - 3\nu_i + \nu_{-i}}{8}$
- This is ½ the monopoly solution

Is tit for tat reaction consistent?

$$\frac{\partial z_i}{\partial z_{-i}} = -\frac{1}{2 + \frac{\partial z_{-i}}{\partial z_i}} = -\frac{1}{2+1} = -\frac{1}{3}$$

No! But profits are higher and people know to play it.

Equilibrium-Consistent Actions

- In consistent conjectural variations both players are playing a follower strategy, of responding to the other players' moves.
- Actions should be consistent with maximizing profits in the next round of play but be consistent with the goal of maximizing profits in the game at equilibrium.

Assumptions that meets the spirit of Cournot

- Reactions as well as quantities are chosen to maximize profit
- Each player takes the other's declared quantity and reaction decisions as given and selects its quantity and reaction to maximize its profits in the game

Player i solves the following

$$\alpha - (2 + r^0_{-i})z_i - z^0_{-i} - \nu_i = 0$$

$$\alpha - (2 + r_i)z^0_{-i} - z_i - \nu_{-i} = 0$$

$$z_i, z_{-i} \geq 0$$

Properties

- Whichever player solves first determines the equilibrium (no tatonnement). Thus, full knowledge of the consequences of actions
- If start from tit for tat, the initiating player does better (tit for tat not an equilibrium with asymmetric costs)
- Two potential equilibria from each starting point
- Different starting points lead to different equilibria

Where do we stand?

- Basic Cournot ok as a benchmark and well studied
- Consistent conjectural variations a dead end—what player would play that game knowing the outcome?
- Closed-loop Cournot games (Allaz Vila and Murphy Smeers) play to the weakness of Cournot (amnesia between stages)
- A fully informed player solves to the equilibrium, not just the optimization. However, that equilibrium is unsatisfying

What next?

- A centerpiece of an analysis must be a model of the process of getting to an equilibrium
  - Starting position
  - State of knowledge and player wisdom
  - Cultural environment
  - Legal and other constraints
- The process is more interesting than the equilibrium
- The behavioral process needs to be grounded in data and logic